BLACK HOLES, GALAXY FORMATION, AND THE $M_{\rm BH} - \sigma$ RELATION

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ABSTRACT

Recent X–ray observations of intense high–speed outflows in quasars suggest that supercritical accretion on to the central black hole may have an important effect on a host galaxy. I revisit some ideas of Silk and Rees, and assume such flows occur in the final stages of building up the black hole mass. It is now possible to model explicitly the interaction between the outflow and the host galaxy. This is found to resemble a momentum–driven stellar wind bubble, implying a relation $M_{\rm BH} = (f_g\kappa/2\pi G^2)\sigma^4 \simeq 1.5\times 10^8\sigma_{200}^4$ M$_\odot$ between black hole mass and bulge velocity dispersion (f_g = gas fraction of total matter density, κ = electron scattering opacity), without free parameters. This is remarkably close to the observed relation in both slope and normalization.

This result suggests that the central black holes in galaxies gain most of their mass in phases of super–Eddington accretion, which are presumably obscured or at high redshift. Observed super–Eddington quasars are apparently late in growing their black hole masses.

Subject headings: accretion – quasars: general – galaxies: formation, nuclei – black hole physics

1. Introduction

It is now widely accepted that the centre of every galaxy contains a supermassive black hole. The close observational correlation (Ferrarese & Merritt, 2000; Gebhardt et al., 2000; Tremaine et al., 2002) between the mass M of this hole and the velocity dispersion σ of the host bulge strongly suggests a connection between the formation of the black hole and of the galaxy itself.

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Recent *XMM–Newton* observations of bright quasars (Pounds et al., 2003a, b; Reeves et al., 2003) may offer a clue to this connection. These observations give strong evidence for intense outflows from the nucleus, with mass rates $\dot{M}_{\rm out} \sim 1{\rm M}_{\odot}$ yr^{-1} and velocity $v \sim 0.1c$, in the form of blueshifted X–ray absorption lines. Simple theory shows that the outflows are probably optically thick to electron scattering, with a photosphere of ~ 100 Schwarzschild radii, and driven by continuum radiation pressure. In all cases the outflow velocity is close to the escape velocity from the scattering photosphere. As a result the outflow momentum flux is comparable to that in the Eddington–limited radiation field, i.e.

$$\dot{M}_{\rm out} v \simeq \frac{L_{\rm Edd}}{c}, \tag{1}$$

where $\dot{M}_{\rm out}$ is the mass outflow rate and $L_{\rm Edd}$ the Eddington luminosity, while the mechanical energy flux is

$$\frac{1}{2}\dot{M}_{\rm out} v^2 \simeq \frac{{L_{\rm Edd}}^2}{2\dot{M}_{\rm out} c^2}. \tag{2}$$

It appears that such outflows are a characteristic of super–Eddington accretion (King & Pounds, 2003). We know that most of the mass of the nuclear black holes is assembled by luminous accretion (Soltan 1982; Yu & Tremaine, 2002). It seems likely that the rate at which mass tries to flow in towards the central black hole in a galaxy is set by conditions far from the hole, for example by interactions or mergers with other galaxies. It is quite possible therefore that super–Eddington conditions prevail for most of the time that the central black hole mass is being built up.

This clearly has important implications for the host galaxy. Unlike luminous energy, a large fraction of a mechanical energy flux like (2) is likely to be absorbed within the galaxy, and must have a major effect. To reach its present mass the black hole in PG1211+143 could have accreted at a rate comparable to its current one for $\sim 5 \times 10^7$ yr. During that time, an outflow like the observed one could have deposited almost 10^{60} erg in the host galaxy. This exceeds the binding energy $\sim 10^{59}$ erg of a bulge with 10^{11} ${\rm M}_{\odot}$ and $\sigma \sim 300$ km s^{-1}.

Accordingly it is appropriate to revisit some ideas presented by Silk and Rees (1998., henceforth SR98) and also considered by Haehnelt et al. (1998), Blandford (1999) and Fabian (1999). These authors envisage a situation in which the initial black holes formed with masses $\sim 10^6{\rm M}_{\odot}$ before most of the stars. Accretion on to these black holes is assumed to produce outflow, which interacts with the surrounding gas. Without a detailed treatment of the outflow from a supercritically accreting black hole, SR98 used dimensional arguments to suggest a relation between M and σ. However this still has a free parameter. Given the simple relation (1) one can now remove this freedom. The situation turns out to resemble a

momentum–driven stellar wind bubble. Modelling this gives an $M_{\rm BH} - \sigma$ relation devoid of free parameters, and remarkably close to the observed relation.

2. Black Hole Wind Bubbles

I follow SR98 in modelling a protogalaxy as an isothermal sphere of dark matter. If the gas fraction is $f_g = \Omega_{\rm baryon}/\Omega_{\rm matter} \simeq 0.16$ (Spergel et al., 2003) its density is

$$\rho = \frac{f_g \sigma^2}{2\pi G r^2} \tag{3}$$

where σ is assumed constant. The gas mass inside radius R is

$$M(R) = 4\pi \int_0^R \rho r^2 {\rm d}r = \frac{2 f_g \sigma^2 R}{G} \tag{4}$$

I assume that mass flows towards the central black hole at some supercritical rate $\dot M_{\rm acc}$. The results of King & Pounds (2003) suggest that this will produce a quasi–spherical outflow with momentum flux given by (1). Note that this momentum rate is independent of the outflow rate $\dot M_{\rm out} = \dot M_{\rm acc} - \dot M_{\rm Edd}$ since the outflow velocity v adjusts as $\dot M_{\rm out}^{-1}$ to maintain the relation (1) (King & Pounds, 2003).

The wind from the central black hole will sweep up the surrounding gas into a shell. As is well known from the theory of stellar wind bubbles (e.g. Lamers & Casinelli 1999) this shell is bounded by an inner shock where the wind velocity is thermalized, and an outer shock where the surrounding gas is heated and compressed by the wind. These two regions are separated by a contact discontinuity. The shell velocity depends on whether the shocked wind gas is able to cool ('momentum–driven' flow) or not ('energy–driven' flow). In the absence of a detailed treatment of a quasar wind, SR98 appear to have assumed the second case. In fact for the supercritical outflows envisaged here the first case is more likely, as the argument below shows.

3. Cooling the Wind Shock

The Compton cooling time of an electron of energy E is

$$t_{\rm C} = \frac{3 m_e c}{8\pi \sigma_{\rm T} U_{\rm rad}} \frac{m_e c^2}{E} \tag{5}$$

where m_e is the electron mass and

$$U_{\rm rad} = \frac{L_{\rm Edd}}{4\pi R^2 c b} \tag{6}$$

is the radiation density at distance R from the black hole, and $b \lesssim 1$ allows for some collimation of the outflow. The electron energy E in the postshock wind gas is $\simeq 9m_p v^2/16$, where v is the wind velocity and m_p the proton mass. Combining this with the usual definition

$$L_{\rm Edd} = \frac{4\pi G M_{\rm BH} c}{\kappa} \tag{7}$$

of the Eddington luminosity for black hole mass $M_{\rm BH}$ shows that

$$t_{\rm C} = \frac{2}{3}\frac{cR^2}{GM}\left(\frac{m_e}{m_p}\right)^2\left(\frac{c}{v}\right)^2 b \simeq 10^5 R_{\rm kpc}^2 \left(\frac{c}{v}\right)^2 b M_8^{-1}\ {\rm yr} \tag{8}$$

where $R_{\rm kpc}$ is R measured in kpc and $M_8 = M_{\rm BH}/10^8{\rm M_\odot}$. Clearly this is extremely short for small R, so the flow is efficiently cooled and thus momentum driven at least initially. I note that Ciotti & Ostriker (1997, 2001) emphasize the importance of Compton heating and cooling on quasar inflows and outflows.

The momentum–driven assumption breaks down once $t_{\rm C}$ becomes of order the flow time $t_{\rm flow} = R/v_s$ where v_s is the shell velocity. We can use the momentum–driven shell velocity v_m derived in eqn (14) below to estimate

$$t_{\rm flow} = 8 \times 10^6 R_{\rm kpc}\sigma_{200} M_8^{-1/2}\ {\rm yr} \tag{9}$$

where $\sigma_{200} = \sigma/(200\ {\rm km\ s^{-1}})$. The assumption of efficient cooling is valid out to a radius R_c given by setting $t_{\rm C} = t_{\rm flow} = 1$. We find a total swept–up mass

$$M(R_c) = 1.9 \times 10^{11}\sigma_{200}^3 M_8^{1/2}\left(\frac{v}{c}\right)^2 b^{-1}\ {\rm M_\odot} \tag{10}$$

at this point. Once the shell reaches radii larger than R_c the shocked wind is no longer efficiently cooled, and its thermal pressure accelerates the shell of swept–up gas to a higher velocity $v_e > v_m$ (energy–driven flow) after a sound–crossing time $\sim R_c/v$.

4. The $M_{\rm BH} - \sigma$ relation

I now estimate the speed v_m of the momentum–driven shell by the standard wind bubble argument. At sufficiently large radii R the swept–up shell mass $M(R)$ is much larger than the wind mass, and the shell expands under the impinging wind ram pressure ρv^2 (this characterizes momentum–driven flows; in an energy–driven flow the thermal pressure of the shocked wind gas is dominant, while in a supernova blast wave the momentum injection is instantaneous rather than continuous). The shell's equation of motion is thus

$$\frac{{\rm d}}{{\rm d}t}\left[M(R)\dot R\right] = 4\pi R^2 \rho v^2 = \dot M_{\rm out} v = \frac{L_{\rm Edd}}{c} \tag{11}$$

where we have used first the mass conservation equation for the quasar wind, and then (1) to simplify the rhs. Integrating this equation for $\dot{R}$ with the final form of the rhs gives

$$M(R)\dot{R} = \frac{L_{\rm Edd}}{c}t \tag{12}$$

where I have neglected the integration constant as $M(R)$ is dominated by swept–up mass at large t. Using (4) for $M(R)$ and integrating once more gives

$$R^2 = \frac{GL_{\rm Edd}}{2f_g\sigma^2 c}t^2, \tag{13}$$

where again we may neglect the integration constant for large t. We see that in the snowplow phase the shell moves with constant velocity $v_m = R/t$, with

$$v_m^2 = \frac{GL_{\rm Edd}}{2f_g\sigma^2 c}. \tag{14}$$

We note that this velocity is larger for higher $L_{\rm Edd}$, i.e. higher black hole mass. This solution holds if the shell is inside the cooling radius R_c; outside this radius the shell speed eventually increases to the energy–driven value v_e, which also grows with $M_{\rm BH}$.

I now consider the growth of the black hole mass by accretion. Initially the mass is small, inflow is definitely supercritical, and even the energy–driven shell velocity would be smaller than the escape velocity σ. No mass is driven away, and accretion at a rate $\dot{M}_{\rm Edd}$ can occur efficiently. However as the black hole grows, we eventually reach a situation in which $v_e > \sigma > v_m$. Further growth is now only possible until the shell reaches R_c, and then only until the point where $v_m = \sigma$. Thus given an adequate mass supply, e.g. through mergers, the final black hole mass is given by setting $v_m = \sigma$ in (14). Using (7) we find the relation

$$M_{\rm BH} = \frac{f_g}{2\pi}\frac{\kappa}{G^2}\sigma^4 \simeq 1.5\times 10^8\sigma_{200}^4\ {\rm M_\odot}. \tag{15}$$

This is remarkably close to the observed relation (Tremaine et al., 2000).

Presumably most of the swept–up mass ends up as bulge stars, and we may tentatively identify $M(R_c)$ as an upper limit the bulge mass M_b of the galaxy. Using (15) to eliminate σ_{200} we get

$$M_{\rm BH} \gtrsim 7\times 10^{-4} M_8^{-1/4}\left(\frac{c}{v}\right)^2 bM_b. \tag{16}$$

If c/v (determined by the ratio $\dot{M}_{\rm out}/\dot{M}_{\rm Edd}$) attains similar values at this point in most systems and the swept–up mass is close to $M(R_c)$ one gets a relation between black hole and bulge mass of the form $M_b \propto M_{\rm BH}^{1.25}$. The relation is written instead in the form (16) to allow easy comparison with the correlation found by Magorrian et al. (1998). Evidently this is not as clear–cut a relation as that between $M_{\rm BH}$ and σ, and indeed the scatter in the observed relation is considerably larger.

5. Discussion

The $M_{\rm BH} - \sigma$ relation given here has no free parameter. If the outflow velocity v had been larger by an optical depth factor $\tau > 1$ (i.e. most of the acceleration occurs below the photosphere) a factor $1/\tau$ would appear on the rhs. However this would require outflow velocities $\tau(GM/R_{\rm ph})^{1/2}$ larger by the same factor than those observed in supercritically accreting quasars.

The lack of freedom in (15) comes about because the physical situation envisaged by SR98 and also studied by Haehnelt et al., 1998) and Blandford (1999) can now be made more precise: the response of observed black hole systems to super–Eddington accretion appears to be an optically thick outflow driven by continuum radiation pressure. Fabian (1999) considers *sub*–Eddington accretion, but emphasizes the importance of the momentum of the outflow as opposed to its energy: it is this which leads to the σ^4 dependence rather than σ^5. Specifically, Fabian (1999) assumes a wind of speed v_w with mechanical luminosity a fixed fraction a of $L_{\rm Edd}$. This produces a relation of the form (15) but with an extra factor v_w/ac on the rhs; it therefore reduces to (15) if one assumes $a \sim v_w/c$. Parameters also appear in other derivations using different physics, such as the ambient conditions in the host galaxy (Adams, Graff & Richstone, 2001) or accretion of collisional dark matter (Ostriker, 2000).

The picture presented here invokes a largely spherical geometry for the ambient gas, except that the accreting matter must possess a small amount of angular momentum to define an accretion disc plane and thus a small solid angle where inflow rather than outflow occurs. It is therefore appropriate to the growth of a spheroid–black hole system. However once most of the gas lies in the plane of the galaxy the momentum–driven outflow considered here would not halt inflow. Evidently this means that accretion from this point on adds little mass to the hole.

If the derivation of the $M_{\rm BH} - \sigma$ relation given here is some approximation to reality, it implies that the central black holes in galaxies gain most of their mass in phases of super–Eddington inflow. As relatively few AGN are observed in such phases, these must either be obscured (cf Fabian, 1999) or at high redshift. It appears then that those quasars which are apparently now accreting at such rates (Pounds et al., 2003a,b; Reeves et al., 2003) are laggards in gaining mass. This idea agrees with the general picture that these objects – all narrow–line quasars – are super–Eddington because they have low black–hole masses, rather than unusually high mass inflow rates.

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